28th ANNUAL UNIVERSITY OF MARYLAND
HIGH SCHOOL MATHEMATICS COMPETITION
PART II
November 29, 2006, 1:00–3:00

NO CALCULATORS
2 hours

1. In this problem, a half deck of cards consists of 26 cards, each labeled with an integer from 1 to 13. There are two cards labeled 1, two labeled 2, two labeled 3, etc. A certain math class has 13 students. Each day, the teacher thoroughly shuffles a half deck of cards and deals out two cards to each student. Each student then adds the two numbers on the cards received, and the resulting 13 sums are multiplied together to form a product P. If P is an even number, the class must do math homework that evening. Show that the class always must do math homework.

2. Twenty-six people attended a math party: Archimedes, Bernoulli, Cauchy, ..., Yau, and Zeno. During the party, Archimedes shook hands with one person, Bernoulli shook hands with two people, Cauchy shook hands with three people, and similarly up through Yau, who shook hands with 25 people. How many people did Zeno shake hands with? Justify that your answer is correct and that it is the only correct answer.

3. Prove that there are no integers $m, n \geq 1$ such that

$$\sqrt{m+\sqrt{m+\sqrt{m+...+\sqrt{m}}}} = n,$$

where there are 2006 square root signs.

4. Let c be a circle inscribed in a triangle ABC. Let ℓ be the line tangent to c and parallel to AC (with $\ell \neq AC$). Let P and Q be the intersections of ℓ with AB and BC, respectively. As ABC runs through all triangles of perimeter 1, what is the longest that the line segment PQ can be? Justify your answer.

5. Each positive integer is assigned one of three colors. Show that there exist distinct positive integers x, y such that x and y have the same color and $|x-y|$ is a perfect square.